JET PROPULSION LABORATORY **INTEROFFICE MEMORANDUM**

3349–04–XXX
October 16, 2005

To: GISMO Team
From: E. Rodríguez
Subject: Scaling the GISMO measurement to airborne platforms

Abstract

I examine how the GISMO measurement should best be scaled from the spaceborne measurement to the airborne implementation. The main conclusion for the system parameters is given in the table below:

Parameter	Value
Starting Cross-Track Distance	1 km
Ending Cross-Track Distance	6 km
Interferometric Baseline	10 m–20 m
System Bandwidth	60 MHz–90 MHz

Given conversations with Torry Akins during the last week, it seems that 80 MHz might be a good all-around compromise.

The technique advocated for the GISMO measurement from a spaceborne platform cannot be fully scaled to an airborne demonstration. Full scaling would require scaling the ice thickness and wavelength, along with the other system parameters. However, the depth of the ice sheets cannot be controlled and scaling the wavelength cannot be accomplished without also substantially changing the penetration depth. The intent of this memorandum is to show how the system parameters should be best scaled to demonstrate the essence of the technique, and, more importantly, acquire useful data over the ice sheets.

Preserving the Fringe Rate Separation

The key observation in the the GISMO technique is that the interferometric fringe rate from the basal layer and the ice surface are significantly different, for a range of angles. Using the equations from Rodriguez and Wu (2005), one finds that η, the ratio of the two fringe frequencies as a function of x_b, the cross-track distance measured at the basal layer, is given by

$$\eta = \sqrt{1 + 2\frac{H}{x_b}\frac{nD}{x_b}}$$

where H is the platform height above the ice surface, $n \approx 1.8$ is the ice index of refraction, and D is the ice sheet thickness. To demonstrate the measurement, one should preserve the same range of frequency ratios as for the spaceborne case. This implies that, given the platform height and ice thickness, the basal distance for a given frequency ratio will be given by

$$x_b = \sqrt{\frac{2HnD}{\eta^2 - 1}}$$

This implies that the cross-track distances should scales as

$$\frac{x_b}{x_b'} = \sqrt{\frac{H}{H'}}$$

Notice that this equation is independent of wavelength or interferometric baseline. If one assumes that the spaceborne GISMO measurement covered a 50 km swath from 10 km to 60 km, the by reducing the platform height from 600 km to 6 km (18,000 ft), the cross track swath should span from 1 km to 6 km in the cross track direction. Notice that the range of incidence angles at the surface will scale approximately in inverse proportion to the the square root of the height

$$\frac{\tan\theta}{\tan\theta'} = \sqrt{\frac{H'}{H}}$$

In the case we considered above, the maximum incidence angle would increase from approximately from about 6 degrees to about 45 degrees. This means that in order for the full range of measurements to be possible, one must use a single dipole antenna element to transmit and receive over the entire swath. Alternately, a synthetic beam must be formed by combining the different elements and steering them electronically over the swath **after individually collecting each element**. This requirement translates into a requirement for separately digitizing each antenna element and avoiding hardware element combining.

Preserving the Number of Fringes

In order to mimic the ability the ability to separate the basal and ice surface fringe frequencies, one must require that the basal frequency be resolved to the same level in both measurements. This implies that the number of fringes over the swath must be preserved. The number of fringes over the swath is approximately given by

$$\Delta\Phi = kB\Delta \sin\theta \approx kB\Delta \tan\theta$$

Since we have seen that $\tan\theta$ will scale as the square root of the platform height, and since we wish to preserve the wavelength, this requirement states that the baseline must scale as

$$\frac{B}{B'} = \sqrt{\frac{H}{H'}}$$

For the spaceborne GISMO design, the baseline was at least 45 m, which implies that it must be at least 4.5 m for the airborne design. Since the swath may be somewhat reduced due to the antenna illumination, and since for the airborne measurements it is desirable to demonstrate a higher accuracy, a longer baseline will chosen in practice, so that the baseline will scale as $B \sim \zeta\sqrt{H}$, where ζ is a factor which may range from approximately 2 to 4, depending on the airborne platform.

Preserving the Geometric Correlation

One of the main constraints in the design of a radar interforometer is that the change of interferometric phase over a single range resolution cell must be much less than 1. The relevant ratio that determines the amount of phase wrap over a range cell, aside from constant factors, such as the wavelength, is given by

$$\frac{B}{\Delta f H \tan\theta}$$

where Δf is the system bandwidth. Given the dependence of baseline and incidence angle on height derived above, one must have that in order to retain the same level of correlation or better, the bandwidth must scale as

$$\frac{\Delta f}{\Delta f'} = \zeta \frac{H}{H'}$$

so that keeping the geometric correlation constant would require an increase of at least 100, from 6 MHz to 600 MHz, which is not feasible with the current hardware.

A less stringent requirement is that the number of phase wraps over a pixel must be much less than π, without trying to preserve the same level of geometric correlation. This requirement translates to the following requirement for the bandwidth

$$\Delta f \gg f\frac{B}{x_b}$$

Assuming $f = 450$MHz, $B < 20$ m, and $x_b = 1$ km, then one must have that $\Delta f \gg 9$ MHz. Typically, one desires on the order of a factor of 10, so that $\Delta f \sim 90$ MHz. This could be relaxed by a factor of 2 for shorter baselines, and would be a factor of 2 worse if ping-pong mode were employed.

Preserving the Number of Samples per Swath

An alternate requirement on the system bandwidth is given by the desire to preserve the number of samples per swath. The number of range samples is linearly proportional to the bandwidth. The swath, on the other had goes down as the square root of the height ratios. Therefore, the number of samples per swath will be preserved if the bandwidth also scales as the square root of the height ratio. For the scaling we have been considering, this implies that the bandwidth should be on the order of 60 MHz. Given conversations with Torry during the last week, it seems that a bandwidth of 80 MHz might be a good compromise for a number of reasons.